Mathematics and Technology

Past, Present and Future

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Presenters

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- **Professor of Math/Stat at Radford University, Virginia, USA.**
- **Founder of the *Asian Technology Conference in Mathematics* (ATCM) which is part of Advanced Technology Council in Mathematics, Inc. (www.atcminc.com)**

Radio Frequency Identification

Sung-Chi CHU

- **Current Interests: ePlatform, RFID and AOLA**
- **email: sungchichu@atcminc.com or sungchi@cuhk.edu.hk**
- **Ph. D. in Computer Science & Applications (1987, Virginia Tech – Hokies, No. 5 on Nov 20)**
- **Was Professor of Computer Science at Radford University, Virginia, USA; now no 9 to 5 job**
- **‘Founder’ of a good Korean Restaurant in Hong Kong, which is not part of ATCM, Inc.**

Outlines

- Past – **Bring forth issues and concerns**
  - **to enable mathematics learning with technology**
- Present – **How are we doing?**
  - **Mathematics + Technology/Technology + Mathematics**
- Future – **Mathematics for ALL**
  - **Do FACT with AOLA facilitation**
- **Concluding Remarks – Action Plans**
  - **Mathematics & mathematics education communities**
  - **Technology landscape**
  - **Collaboration: teachers, institutions and …**

Maths and Tech…is Evolving as Evident…

- "Computer Technology in Mathematical Research and Teaching" ***ATCM 95, 97, 98***
- "Applications of Technology in Mathematics Research and Teaching for the 21st Century" ***ATCM 99, 00, 01***
- "Multimedia for the Advancement of Mathematics" ***ATCM 02***
- "Technology Connecting Mathematics" ***ATCM 03***
- "Technology in Mathematics: Engaging Learners, Empowering Teachers and Enabling Research" ***ATCM 04***
- "Enriching Technology in Enhancing Mathematics for All" ***ATCM 05***

Past

(look back 10 years – Pre-ATCM)

- To create visualization with ease by using graphing calculator
- To claim glory-glory when you figure out how to key in a complex equation and a textual graph is displayed
- Computer is a luxury
- Software is like an LV bag – you cannot afford even ONE, and fake/copy is illegal and not perfect

Present Time
(today)

- Go 3D if you want
- Colors no longer thrill
- You know LaTeX and I know Scientific Notebook, but we do not know MathML
- *Labs* of computers
- The choice of software or calculators is what you can have, but yet deciding which one is still a mystery:
  - Don’t worry, there is room for everyone.
  - There is no software that can do everything.

Present Time

(issues and concerns)

- Sharing is a problem (interoperability)
- On-demand visualization cannot be taken for granted (dynamicity)
- The Web is IT (connectivity, interactivity)
- Languages for maths: Equation syntax varies from one software to another.
- Collaboration needs to go global and stronger (integrativity);
- Technology should be used as a bridge to link all areas that are related to mathematics?

Present Time

Gains and improvements…

- Abstract and complicated concepts can be learned assisted with technology (examples given later)
- Expanding students' learning horizon (example – fusion of Geometry with Calculus; 2D goes 3D)
- Extending researchers' reach (example - make conjectures)
- Mathematics is more accessible through/with technology

What Should We Do Next?

- How to go about doing it?
- We have some ideas to share
  - Wei-Chi: FACT
  - Sung-Chi: AOLA
  - The Concluding Chi's: FACTAOLA

True or False?

Exploring Mathematics with Technology

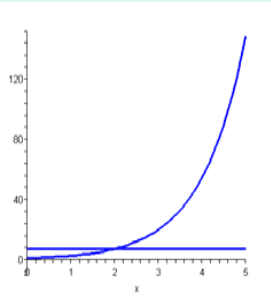
Should be (a)

F-A-C-T.

Mathematics should be *Fun*

past, present, and future

- Fish-maple file (Thanks to Ebisui from Japan).
- Hat-maple file (Thanks to Ebisui from Japan).
  - Journal file.
- Mathworld.wolfram.com-offline
- Easter Egg.
- A donut, a yo-yo, a sphere and etc.
  - (152f01->revol.mws)
- Height of the Pole (Java applet)
  - (A Casio file->telephonepoles)



Middle school: About angles.

- (CP->lineartransform->aboutangles.fls)

Accessible-part 1

When Geometry is integrated with CAS (present)

- **Derivative functions by animation (classpad->derivative-sine)**
- **Derivative at one point. (classpad->slopeoftangent.fls) (maple file)**
- **Shortest distance (classpad->shortestdistance.fls)**
- **Mean-Value Theorem. (classpad->roles.fls)**
- **A Rope (cp2); Journal file**
- ***Derivative for an implicit function (classpad->implicitdiff.fls) (use CP2.0)**
- ***Maximum and Minimum problem (classpad->maximumarea.fls) (use CP2.0)**

Accessible-part 2

Impacts of graphs: (ideas are not new but tools are)

- **Java applets for the sin(x), cos(x)-the desktop (Thanks to IES from Japan)**
  - **Basic sine (classpad->basicsine.fls).**
  - **Relate y= arcsin(x) and y=arcos(x) graphically.**
  - **Relate y= arccsc(x) and y=arcsec(x) graphically.**
  - **Relate y=arctan(x) and y=arccot(x) graphically.**
- ***Shiftings, expansions and the graphs of inverse functions. (definenew-cp2); journal file.**
- ***Prove arcsin(x)+arcos(x)=Pi/2. (Journal file) (define new-cp2)**

Simulate infinite series of functions by using finite sequence of functions

- **Fundamental Theorem of Calculus. (Maple)**
- **Nowhere differentiable functions. (journal file; nowherediff.mws)**
- **Taylor series-local approximation**
- **Fourier Series-global approximation (link->fourier.rap); Gibbs**

Accessible-part 3

Abstract and complex concepts can be understood by more people.

- The epsilon-delta problem (Journal file) (Maple)
- About quantifiers. (journal file) (swp)
- Uniform Convergence, uniform continuous functions (Maple)
- Linear Transformations:
  - 2d (IES)
  - 3d (internet needed)
  - CP->transform.fls

Challenging-part 1

- **Shrinking circle-James Stewart's text book,**
  - **2d (classpad->shrinking.fls).**
  - **Gif file (thanks to Douglas Meade)**
  - **An ellipse (gsp) (Thanks Nicholas Jackiw and Daniel Scher)**
  - **A generalization (by Robert Kreczner)**
    - **Numerical computation fails**
- ***Probability and calculus problem from 2d to 3d.**
  - **2d (classpad->probability.fls)**
  - **3d**
- **Foxes and Rabbit problem: iteration on matrices.**
- **Donkey problem. (a journal file)**
- **Where to sit in a movie theater (from James Stewart textbook), 2d and 3d. (journal file), (maple file)**

Challenging-part 2

Present and Future

- 3D geometry will impact us (Professor J.C. Chuan's web page)
  - Example 1
  - Example 2
- Research in many pure mathematics areas can be understood and expanded thanks to technology.

Theoretical-last

- =>VERIFY=> NEW THEOREMS.
- EXPERIMENT=>CONJECTURES
  - Traditionally: Theorem->Prove->Homework.
- Existence of a solution is not good enough=> Where is the solution?
- Graphical Simulation
- Iterations on numbers or matrices
- Geometry in 2D and 3D + CAS.

Questions

- Is there one software that can show all? Probably yes, but you still need the many brains that create each individually together
- Can you show all these on the Web? Most likely
- Can we take your Maple example and recreate it in Mathematica? In graphing calculators?
- Why don't I ask the Korean expert to clone you and I will travel with you in one big LV luggage?

Technology + Mathematics

= AOLA

Yeah? When?

Where are we with ‘Technology’?

- The computer is good – help in learning Mathematics?
  - tools; games; pay bills without leaving the house; same page
- The Web is intensive – help in enhance learning in Mathematics?
  - cyber space; ‘resources’; search engine
- The mathematical software is powerful – help in visualization at least
- What is missing?
  - Information standards; communication platform;

Current Practices

- How did we get here?
  - Personal computers: faster and affordable
  - Web: common interface & unlimited and unrestricted “resources”
  - Mathematics software: characters to color pixels, drag-and-view (?), …
  - Information standards: HTML, XML, MathML, SOAP, …

Current Practices

- Different environments
  - Standalone: desktops/laptops/hand-held
  - Web-based & Wireless access
- Different levels of dynamicity
  - Static – viewable only
  - Interactive – interactively viewable
  - Dynamic – interactively viewable with manipulation

What is missing?

Tell me what you need with respect to “learning mathematics in an electronic environment”, then I will to tell you what is missing.

Learn Mathematics

Web-based

Enabled for the Web

What do we need with respect to mathematics education?

- Enhance the usability of technology in mathematics learning
- Enhance the explorability of mathematics activities
- Enhance the interactivity of teachers and learners
- Enhance the measurability of the performance of both the learners and the teachers

An e-Platform Framework for Mathematics Learning

- '*e*' must be "*integrative*"
- Platform must enable "*collaborative* learning"
- Mathematics content must be AOLA – author-once-learn-anywhere
- Learning must be enabled for anywhere, anytime, and *anyone*

Framework: Integrative

- Connectivity - linking of and among users to linking of resources
- Interactivity – request-response paradigm must be extended to any-to-any
- Interoperability – at different levels such as data, information and activity
- Integrability – manage all in one
- Security – assessment and personal effects

Framework: E-Collaboration

- **Communication**: exchange of textual and/or graphical contents between parties
- **Contribution**: exchange of contents that are beneficial to the other parties
- **Coordination** & **Cooperation**: reciprocal exchanges to coordinate resources versus a common task that otherwise would not be able accomplished alone
- **Collaboration**: requires joint intellectual efforts

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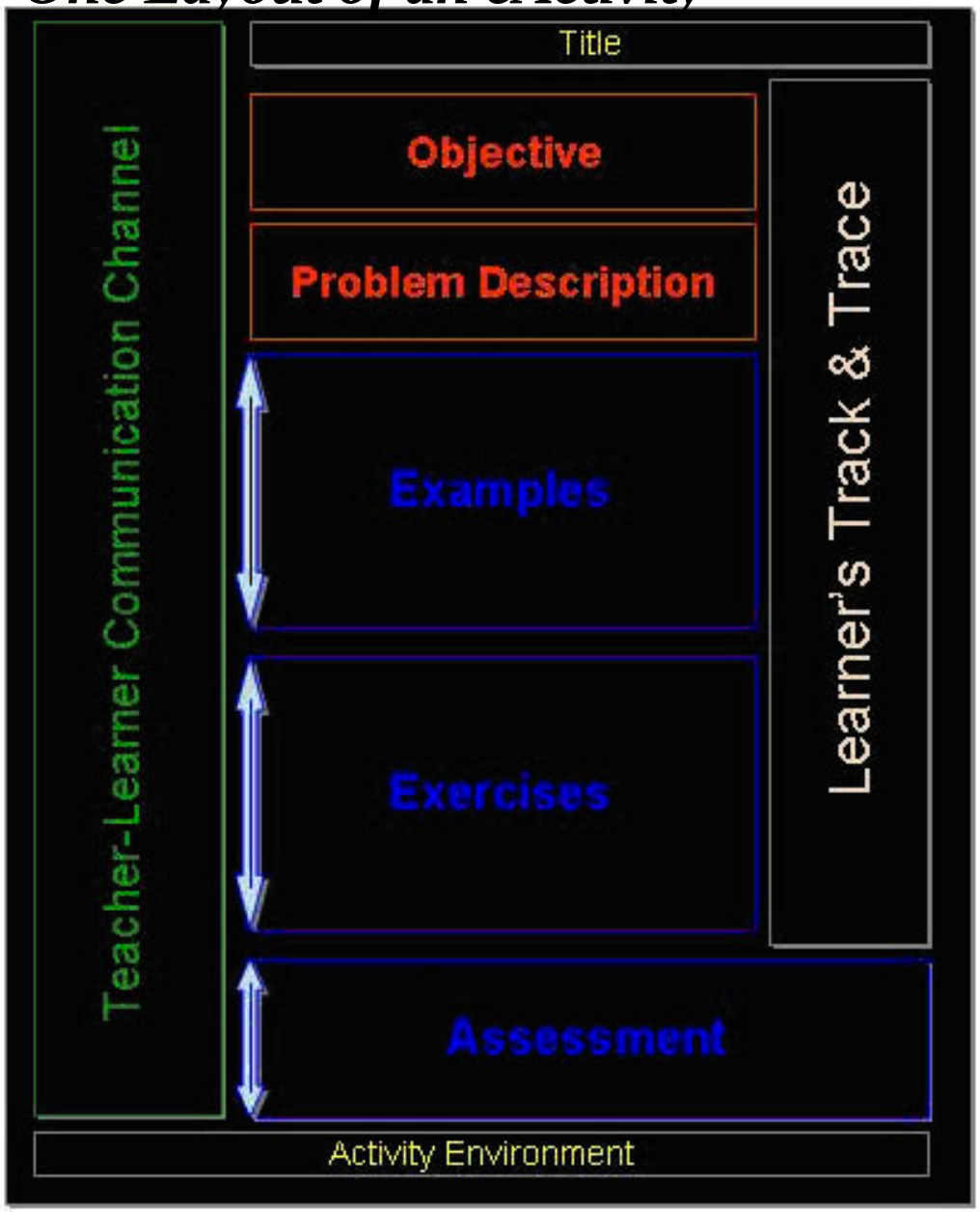
Mathematics Content: An *e*Activity

- Level of learners
- Objectives – what will they experience & learn
- Problem domain and statement
  - Learning Process
  - Exploratory Process
  - Assessment Process
- ‘activity’ becomes a template – an activity template service

One Layout of an eActivity



OK…

Web-based…

Technology Ready?

How do you put all these ‘things’ in the webpage?

<img *src=*
“http://
www.knue.edu.kr/
***~hclew/picture.jpg*”>**

HTML

XHTML

ActivityM(arkup)L(anguage)

- XML syntax + XSchema – ‘extensible’ and ‘interoperable’
- Goals
  - To capture and distinguish all possible activity components, as simple as activityExpiredDate to activityCommunicationChannel
  - To enable the specification of individual mathematics objects with dynamic behavior
  - To support transformation into/from different existing ‘activity’ formats

An e-Activity Sample in ActivityML

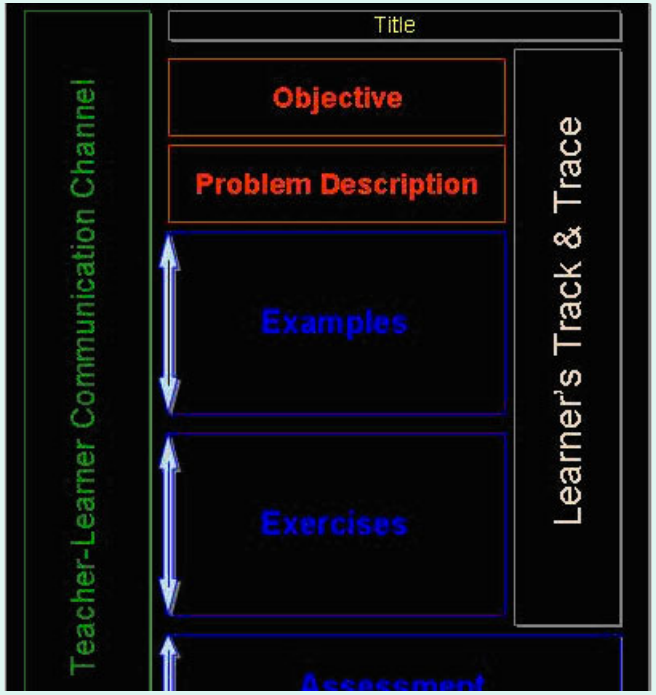
```
<?xml version="1.0" encoding="UTF-8"?>
<activity xmlns:xsi="http://www.w3.org/2001/XMLSchema-instance"
        xsi:noNamespaceSchemaLocation="C:\travellingDocument\Any2Any\AOLA\ActivityML\ActivityMLStandards\MEActivity.xsd"
        privacy="public" dateOfCreation="2005-12-10T09:30:47-05:00" id="activityExample002"
        lessonContext="algebra" assessmentType="non-grade" technology="graphingCalculators"
        copyright="none/opensource/AOLA-License">

  <author url='http://www.sungchichu.com'>S.C. Chu</author>
  <dateOfContribution>October 13, 2005</dateOfContribution>
  <dateOfLastModification>Ocotber 13, 2005</dateOfLastModification>
  <privacyPreference>Moderate</privacyPreference>

  <subjectArea>Algebra</subjectArea>
  <topicalArea>Parabolas</topicalArea>
  <topicalArea>Best Fit Lines</topicalArea>
  <topicalArea>Estimation</topicalArea>
  <myTopicalArea>Fast</myTopicalArea>

  <title>The Intersection of Two Planes</title>

  <activityBackground> Heron of
Alexandria lived somewhere around 75 AD (debate ranges between 150 BC to 250
AD).  He wrote extensively on mathematics and physics.  It is thought that he
was an Egyptian with Greek training.  He wrote for practicality as opposed to
theory.  He provided a scientific foundation for engineering and land surveying.
Heron devised a method for finding the square root of a number, n, that is quite
similar to the process used by computers today.
  </activityBackground>
  ..........
```

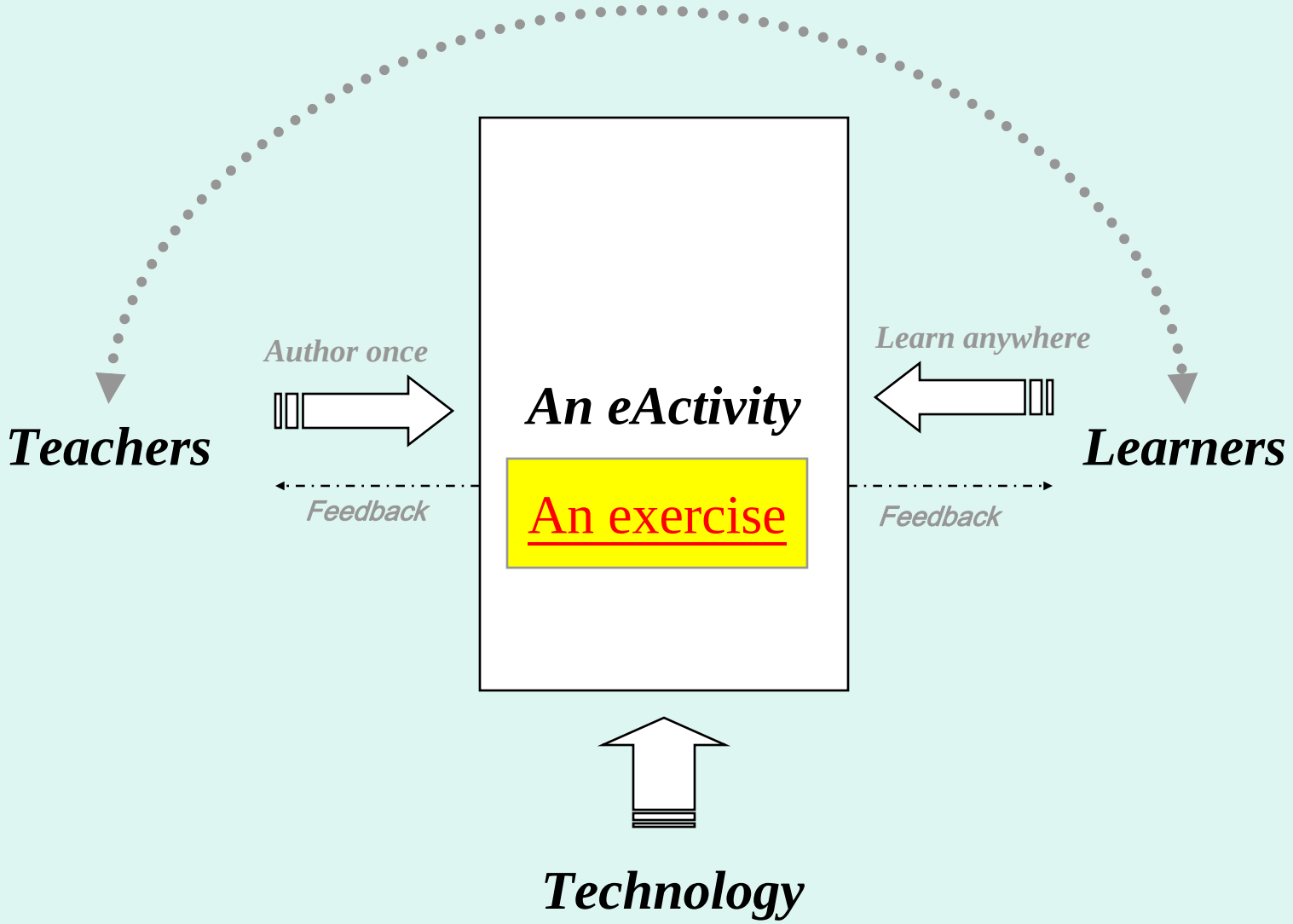

An *e*Activity Sample (cont'd)

```
 <activityDescription> You know the area of a triangle can be expressed as
     <mathObject type="static" representation="LaTeX">A = 1/2 bh</mathObject>.
     You may know about Heron¡¦s formula <mathObject inputMethod="LaTeX"
     type="static">x(s-a)</mathObject> where a, b, and c are sides of the triangle
     and <mathObject type="dynamic" style="detached" inputMethod="maple" scope="explore;modify">s= a+ b +
c</mathObject>
     which also gives the area of any triangle.  There is another formula for the
     area of a triangle.
  </activityDescription>
  <activityObjective>The objective of this activity is to allow the learner to learn about the theorem GreekMyth with
      simple basic exercises.
  </activityObjective>
  <educationLevel>Primary 2</educationLevel>
  <example order="1" tracking="off">1. abc </example>
  <example order="2" tracking="optional">2. mnl </example>
  <example order="3" tracking="required" session="resumable-8">3. xyz</example>

  <exercise order="1"> This is to set the value of a to 3.45 and explore
     <mathObject type="dynamic" representation="applet"
             inputMethod="LaTeX" expressionType="equation" logon="required">a=a+n / 2</mathObject>.  You must
             finish this exercise before you move onto the next lesson.
  </exercise>
  <assessment order="1" tracking="required" session="retricted">
      <reporting>Blackboard</reporting>
  </assessment>

</activity>
```


Rewind

- Mathematics content must be AOLA – author-once-learn-anywhere
- An eActivity
- ActivityML
  - Teachers & learners do not need to know this…



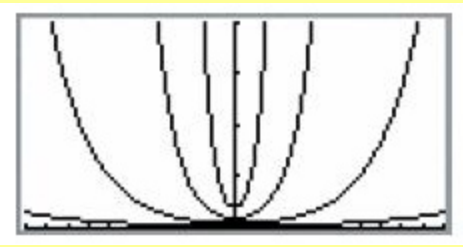

Teachers
Author once
An eActivity
An exercise
Learn anywhere
Learners
Feedback
Feedback
Technology

An exercise

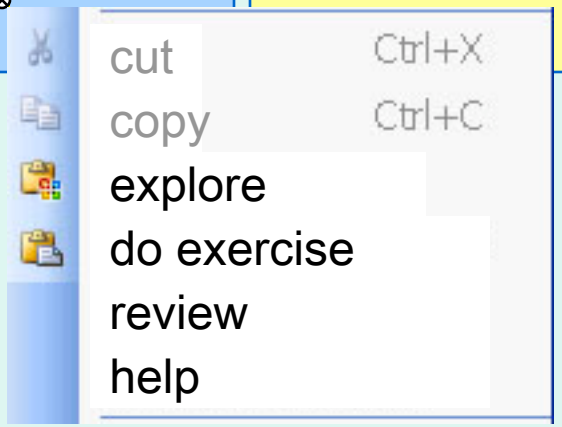
Hold the ends of a chain in each hand.
The chain forms a curve. Which curve is it?

$$y = a\cosh(x/a) = 0.5a(e^{x/a} + e^{-x/a})$$

To everyone's surprise it is not a parabola but a cosine hyperbolic.



Dynamic Mathematics Object



How do you put all these ‘things’ in the webpage?

<img *src=*
“find Prof. Lew Picture”
type=“handsome”>

HTML

XHTML

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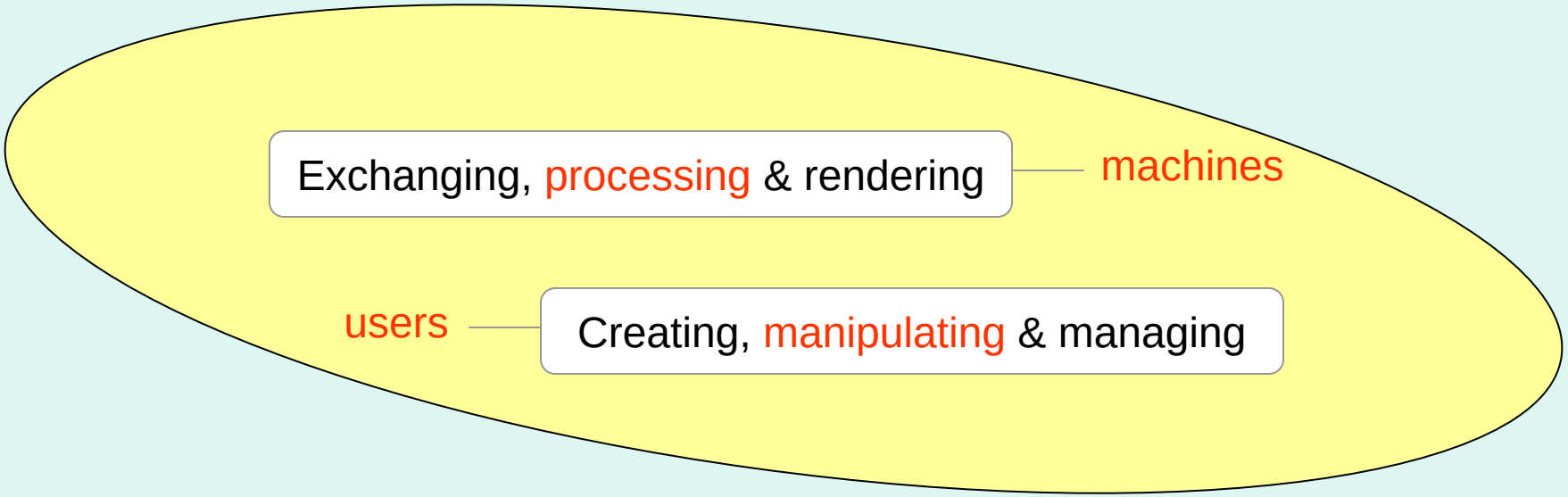
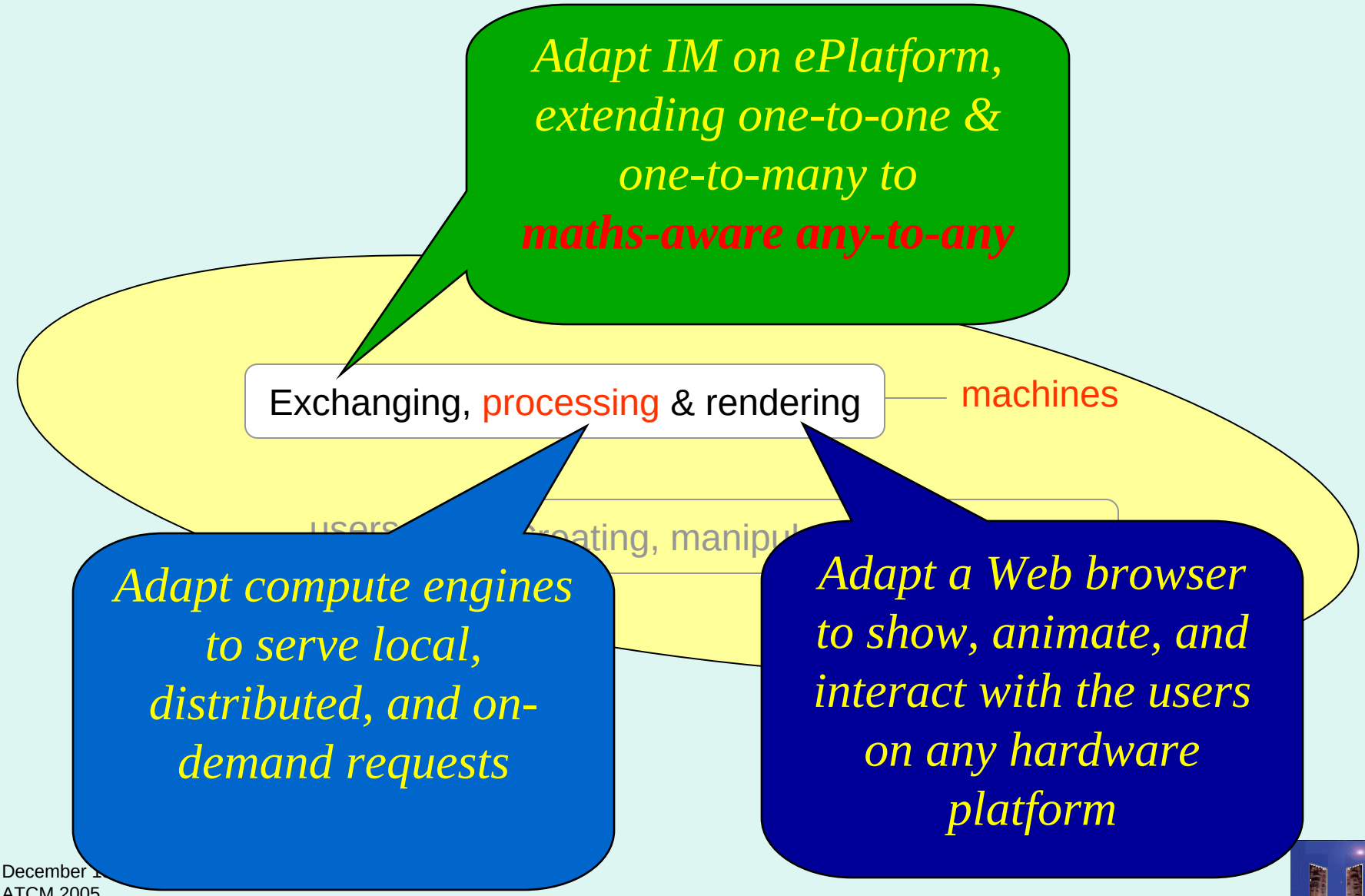
Core Functions in the AOLA ePlatform



Core Functions in the AOLA ePlatform



Core Functions in the AOLA ePlatform

Adapt best practices to a Web-based tool to create activity for deployment in ePlatform – sharable

Adapt best practices to allow management of activity and activity-related resources

Exchanging, p essing & rendering

achines

users

Creating, manipulating & managing

Adapt best practices for enabling Web-based manipulation of mathematics objects – to facilitate explore, learn and assess

The Near Future

Integrate[
[Technology/Mathematics],
Web]
=
?

Integrate[

<mrow><mo>[</mo>
<mfrac><mi>**Technology**</mi><mn>**Mathematics**</mn></mfrac>
<mo>]</mo></mrow],

Web]

=

FACT?*AOLA*

Finally, …

Think AOLA

and

Practice FACT

Thank you very much

- **Kamsahamnida**
- Merci beaucoup
- Danke schön
- Arigato gozaimasu
- Muchas gracias
- Shukran gazilan (Arabic)
- 谢谢 (Xie xie)